Superposition and Spectra

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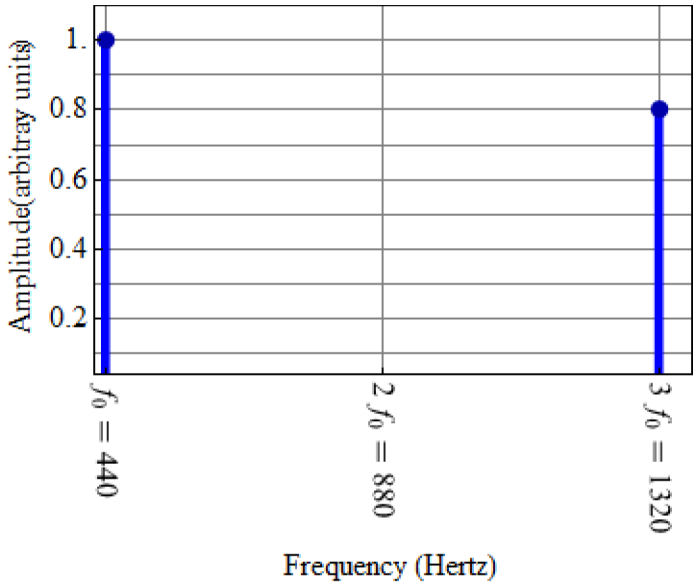
Amplitude Spectrum

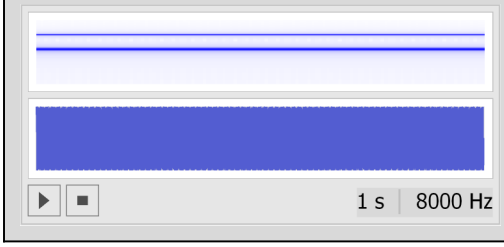
First Example

The graph below shows a spectrum. The meaning of the points with coordinates (**440,1.0**) and (**1320,0.8**) will be explained in this document:



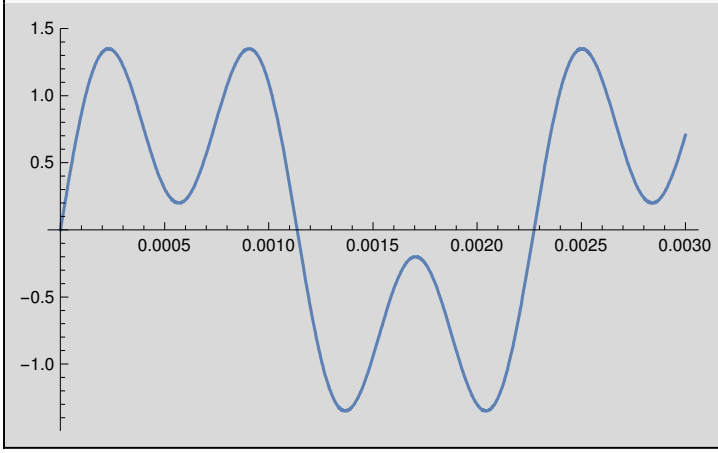
In the previous graph, the point with coordinates (**440,1.0**) means that we have **1.0** multiplying the sine of 2π(**440**)t; then the other point with coordinates (**1320,0.8**) means that we must add **0.8** multiplying the sine of 2π(**1320**)t (if you are reading this document in *Mathematica* or the *CDF-Player*, press the button  in the result of the calculation below):

```
Play[ 1.0 * Sin[2 * Pi * 440 * t] + 0.8 * Sin[2 * Pi * 1320 * t ], {t, 0, 1}]
```

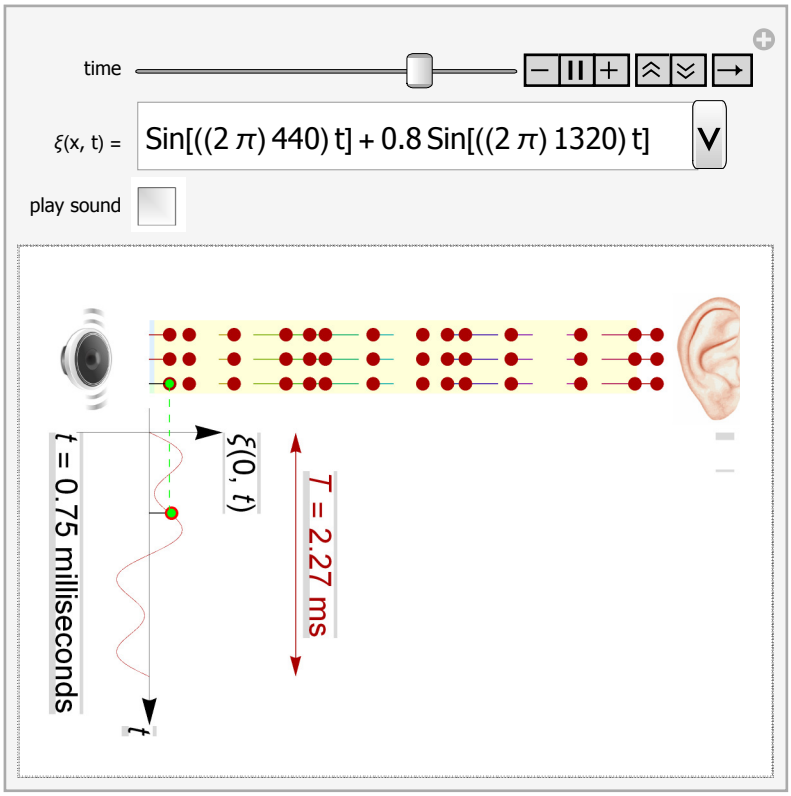


Replace the command **Play** with the command **Plot** and the interval **{t, 0, 1}** (one second) with the interval **{t, 0, 0.003}** (three milliseconds) in order to see the waveform:

```
Plot[ 1.0 * Sin[2 * Pi * 440 * t] + 0.8 * Sin[2 * Pi * 1320 * t ], {t, 0, 0.003}]
```

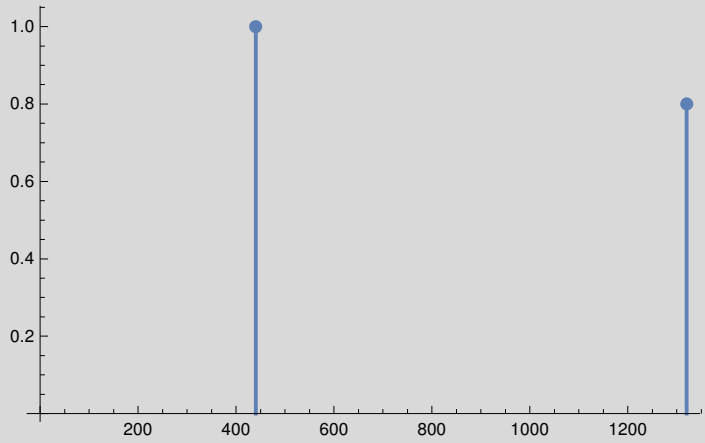


Please **enable dynamic content in *Mathematica* or the *CDFPlayer*** in order to be able to interact with the following demonstration, which is a very simplified model of the motion of air particles following this waveform:



We can obtain a graph of the spectrum with the command ListPlot:

```
ListPlot[
 {{440, 1}, {1320, 0.8}},
 Filling → Axis,
 FillingStyle → Thick,
 AxesOrigin → {0, 0}
]
```



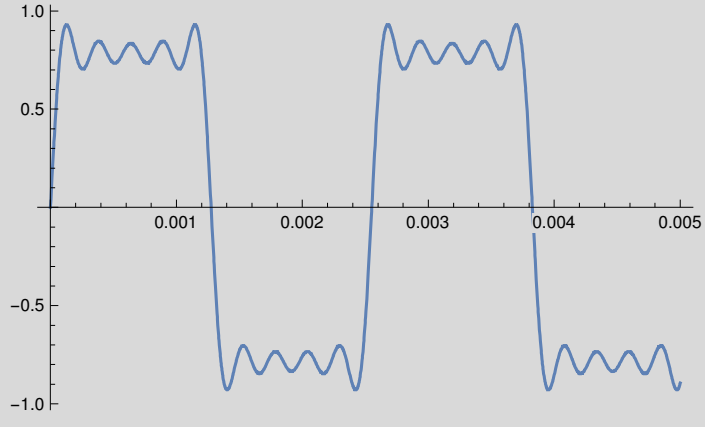
Second Example

We will work with this wavefunction:

$$f(t) = \sin(2 \cdot \pi \cdot 392 \cdot t) + \frac{1}{3}\sin(2 \cdot \pi \cdot 1176 \cdot t) + \frac{1}{5}\sin(2 \cdot \pi \cdot 1960 \cdot t) + \frac{1}{7}\sin(2 \cdot \pi \cdot 2744 \cdot t) + \frac{1}{9}\sin(2 \cdot \pi \cdot 3528 \cdot t)$$

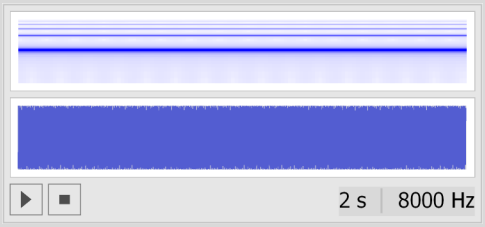
This is the graph (waveform) of the function:

```
Plot[Sin[2 * Pi * 392 * t] + 1/3 * Sin[2 * Pi * 1176 * t] + 1/5 * Sin[2 * Pi * 1960 * t] +
  1/7 * Sin[2 * Pi * 2744 * t] + 1/9 * Sin[2 * Pi * 3528 * t], {t, 0, 0.005}]
```



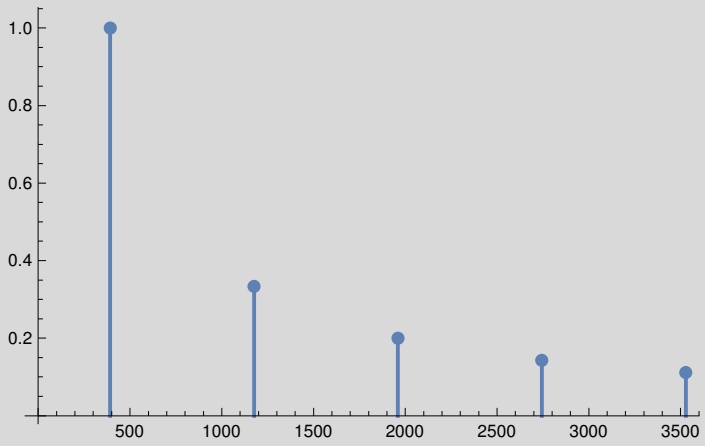
This is the sound of the function (if you are reading this document in *Mathematica* or the *CDF-Player*, press the button ▶ in the result of the calculation)

```
Play[Sin[2 * Pi * 392 * t] + 1/3 * Sin[2 * Pi * 1176 * t] + 1/5 * Sin[2 * Pi * 1960 * t] +
  1/7 * Sin[2 * Pi * 2744 * t] + 1/9 * Sin[2 * Pi * 3528 * t], {t, 0, 2}]
```

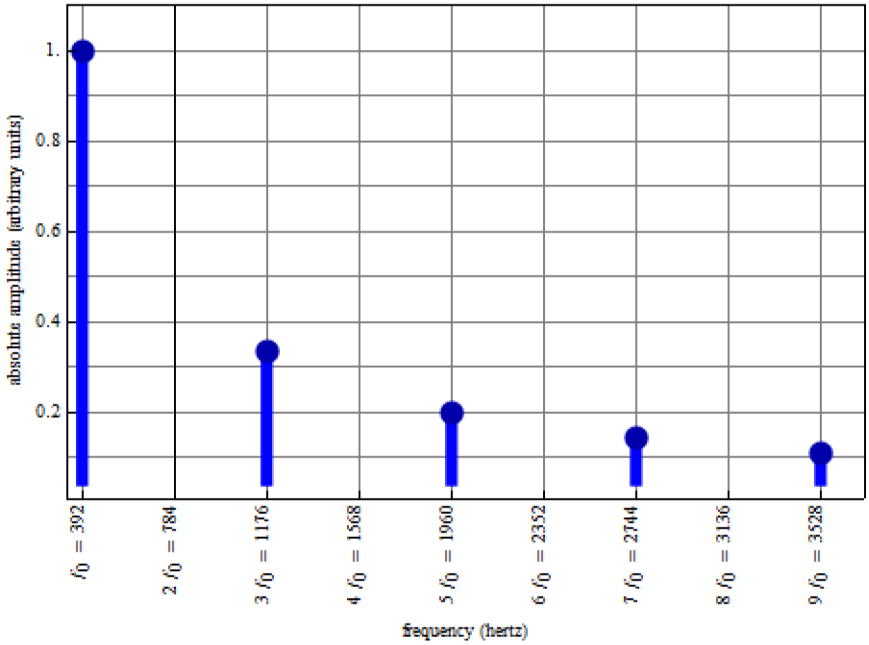


We can visualize the amplitude-spectrum of this wave with the command ListPlot. Remember that every point is made of the frequency and the corresponding amplitude, therefore the expression Sin[2*Pi*392*t] becomes the point {392,1}, the expression $\frac{1}{3} * \texttt{Sin[2 * Pi * 1176 * t]}$ becomes the point $\left\{1176, \frac{1}{3}\right\}$ and so on, as shown below:

```
ListPlot[
 {{392, 1}, {1176, 1/3}, {1960, 1/5}, {2744, 1/7}, {3528, 1/9}},
 Filling → Axis,
 FillingStyle → Thick,
 AxesOrigin → {0, 0}
]
```



The spectrum again, with appropiate labels:



Exercises (Amplitude Spectrum)

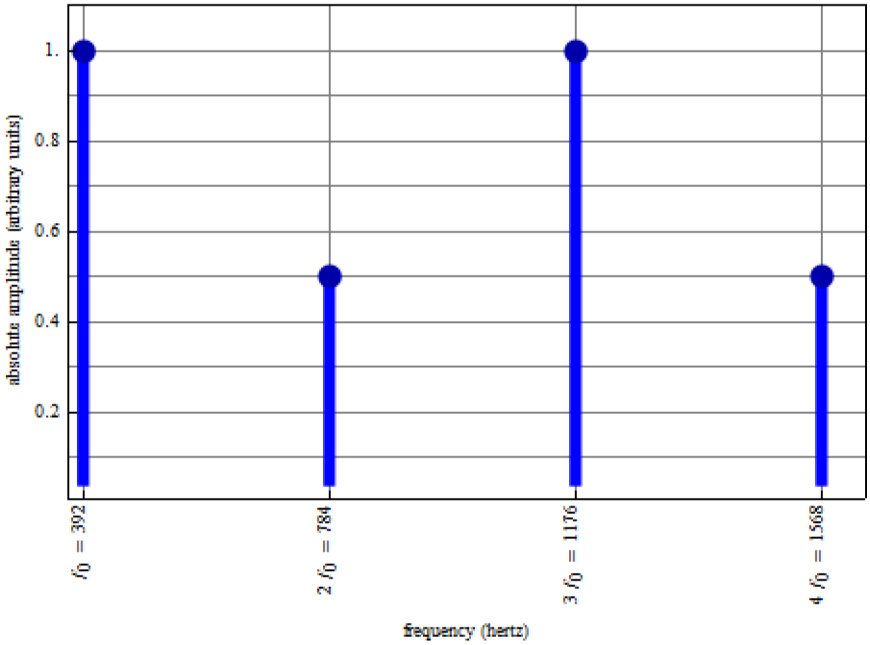
Exercise 1

Obtain the waveform, sound and spectrum for this function:

$$f(t) = \sin(2 \cdot \pi \cdot 523 \cdot t) + \frac{1}{2}\sin(2 \cdot \pi \cdot 1046 \cdot t) + \frac{1}{3}\sin(2 \cdot \pi \cdot 1569 \cdot t) + \frac{1}{4}\sin(2 \cdot \pi \cdot 2092 \cdot t) + \frac{1}{5}\sin(2 \cdot \pi \cdot 2615 \cdot t) + \frac{1}{6}\sin(2 \cdot \pi \cdot 3138 \cdot t) + \frac{1}{7}\sin(2 \cdot \pi \cdot 3661 \cdot t) + \frac{1}{8}\sin(2 \cdot \pi \cdot 4184 \cdot t) + \frac{1}{9}\sin(2 \cdot \pi \cdot 4707 \cdot t)$$

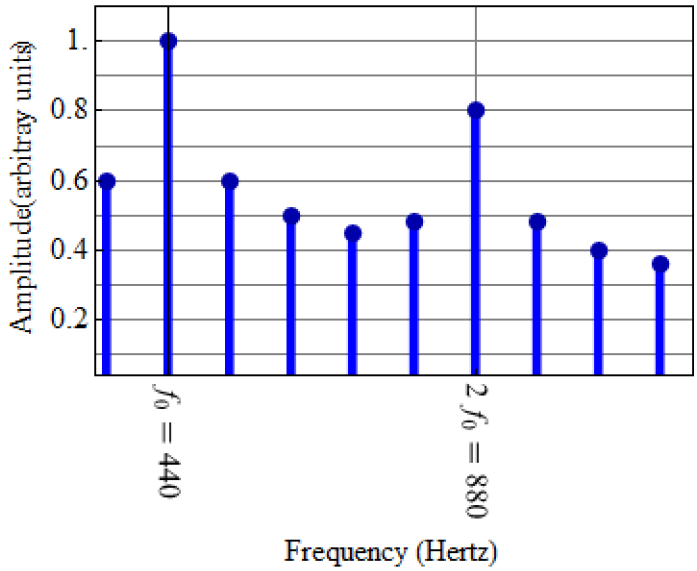
Exercise 2

Obtain the function, waveform and sound for this spectrum:



Exercise 3

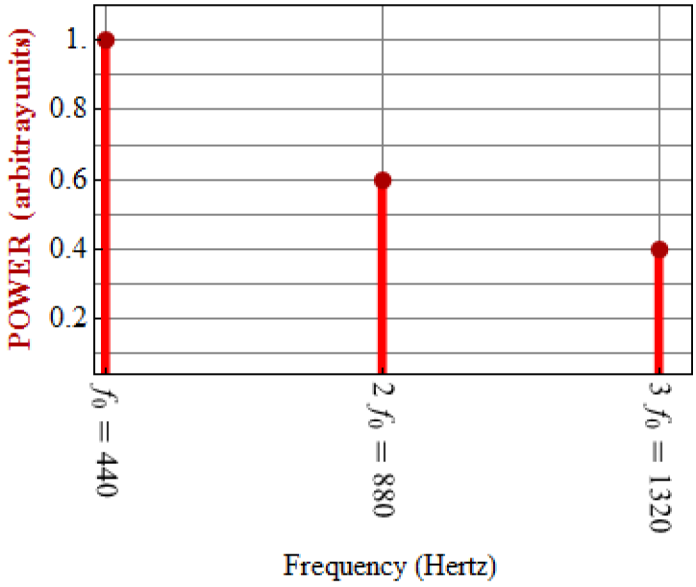
Obtain the function, waveform and sound for this spectrum:



Power Spectrum

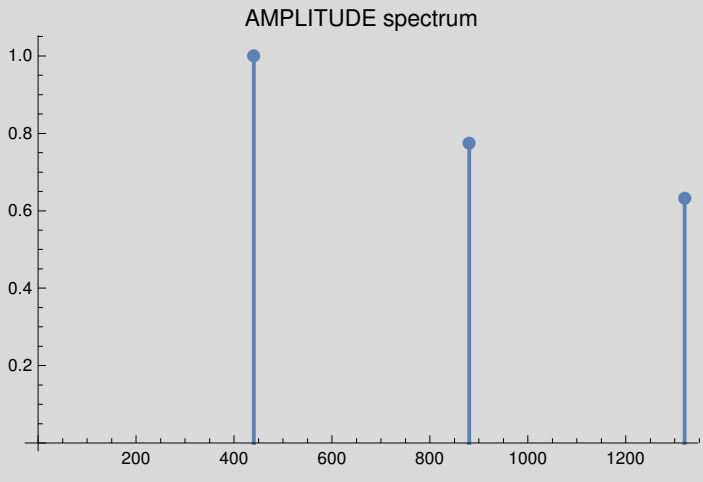
Third Example

Very often the **power spectrum** is used instead of the amplitud spectrum. The difference is that the power is proportional to the **squared magnitude of the amplitude**. Therefore, in the following example, the point with coordinates (**880**,**0.6**) means that we must add $\sqrt{\mathbf{0.6}}$ (notice the **square root**!) multiplying the sine of 2π(**880**)t

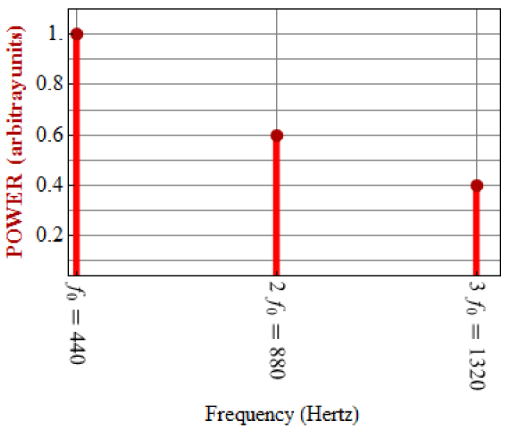


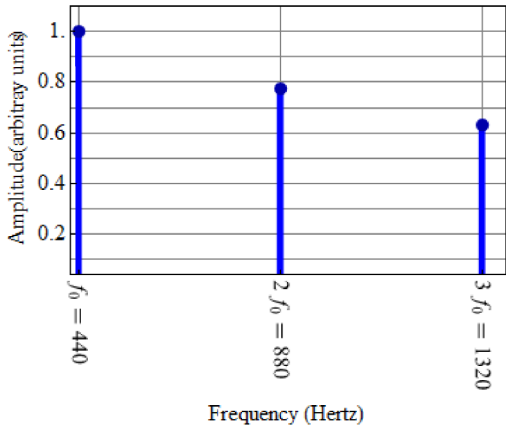
If we want the amplitude spectrum, we must graph the **square roots** of each point:

```
ListPlot[
 {{440, √1}, {880, √0.6}, {1320, √0.4}},
 Filling → Axis,
 FillingStyle → Thick,
 AxesOrigin → {0, 0},
 PlotLabel → "AMPLITUDE spectrum"
]
```



Below we have the power spectrum and the amplitude spectrum of the **same** sound:

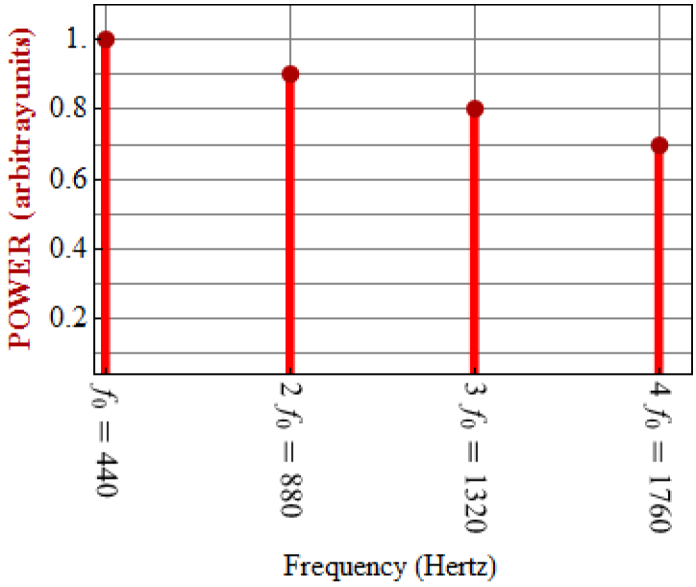




Exercises (Power Spectrum)

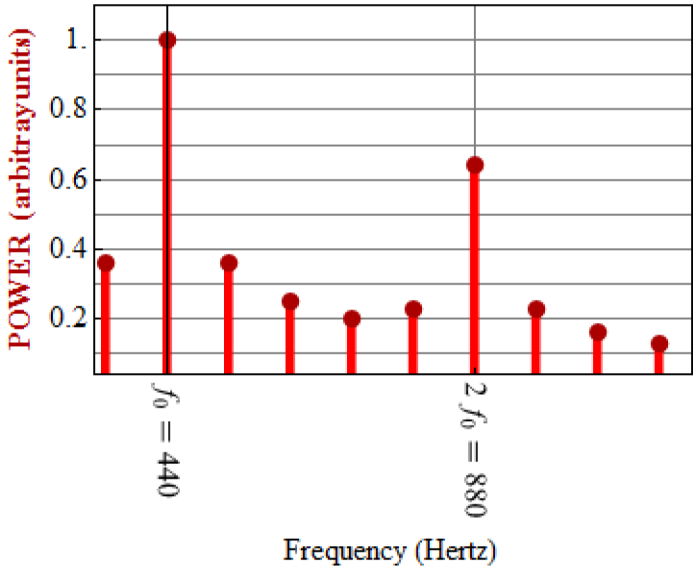
Exercise 4

Obtain the amplitude spectrum of the following power spectrum:



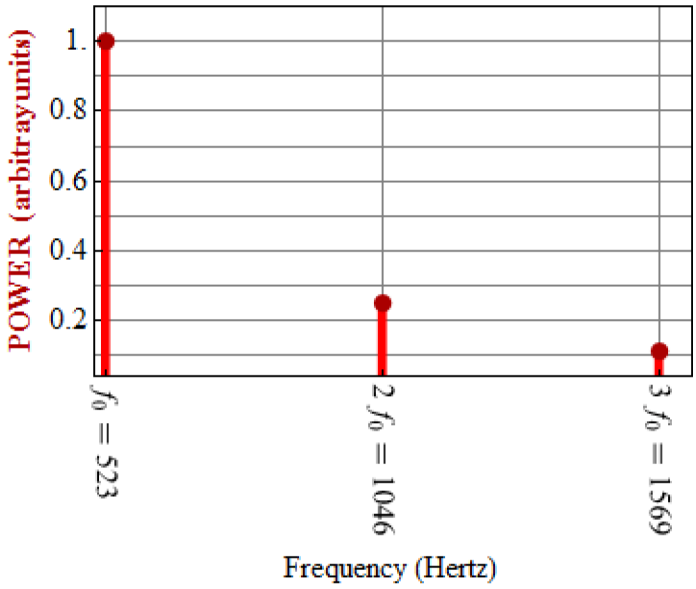
Exercise 5

Obtain the amplitude spectrum of the following power spectrum:



Exercise 6

Obtain the amplitude spectrum, **and from the amplitude spectrum obtain the waveform and sound**, for the following power spectrum:



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