Acoustics of the Recorder

> In dealing with a scientific problem, I first arrange several experiments, since my purpose is to determine the problem in accordance with experience, and then to show why the bodies are compelled so to act.
>
> Leonardo Da Vinci (1452-1519)

OBJECTIVES

To demonstrate some operating features of a woodwind instrument and deduce the physical mechanisms of tone production.

THEORY

For the sake of this exercise, assume that you are the newly-hired house scientist at Execrable Musical Instruments, Ltd. Your goal is to understand enough about their line of cheap recorders to suggest improvements. Being a good scientist, but ignorant of musical instrument technology, you set out to measure the sounds that a recorder produces. Based on the measurements, you can then use a simplified model of the recorder to understand some aspects of tone production in this family of instruments.

At an elementary level, traditional musical instruments consist of a source of mechanical energy, a closely coupled vibrating structure that modifies and radiates the sound and a mechanism for changing the dominant pitch of the emitted tone. Here we will be particularly concerned with the flute-like instruments, a family that includes the transverse flute, piccolo,

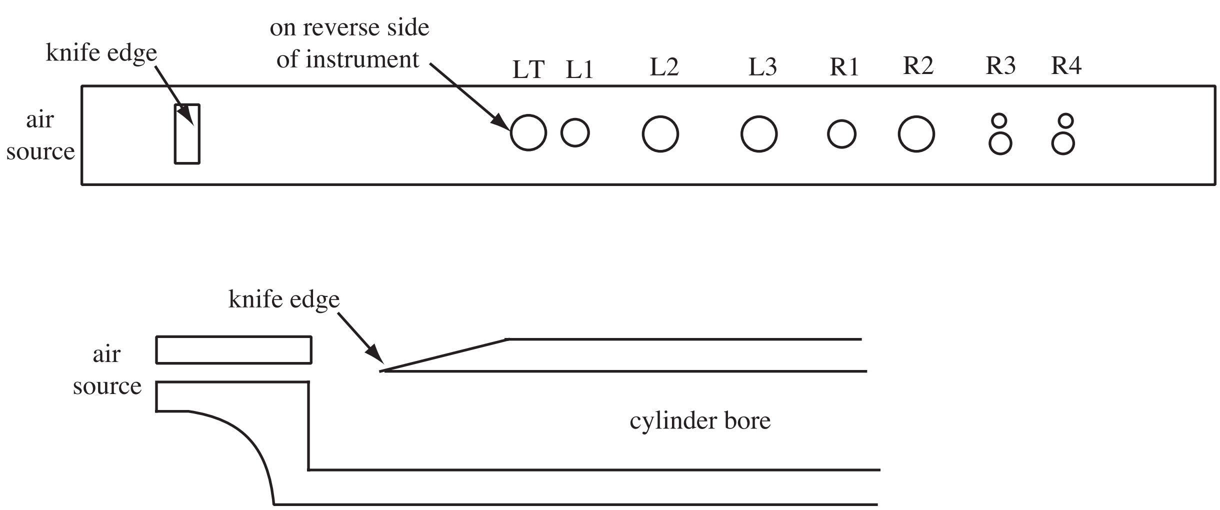

Fig. 1 Top view of a recorder, with an enlarged cross-section of the air inlet end. The player sounds a note by covering some or all of the holes with their fingers and blowing through the end of the instrument.

recorder and pan pipes. The common factors in these instruments include the use of an air jet impinging on an edge as the energy source and a conical or cylindrical air column as the vibrating structure. Pitch control is provided by opening or closing various openings called finger holes along the length of the tube. Skilled players can also use the air flow rate for finer adjustments. The various structures are identified for a recorder in Fig. 1

With all the finger holes closed, the recorder appears to be a slightly tapered cylinder, obviously open at one end and perhaps both ends. Standing waves in the air column occur for wavelengths such that a pressure minimum occurs at an open end or a pressure maximum at a closed end. The expected frequencies are:

$$f_n = n\frac{v}{2L_{eff}} \qquad n = 1,2,3\ldots \text{ open both ends} \qquad (1)$$

$$f_n = n\frac{v}{4L_{eff}} \qquad n = 1,3,5\ldots \text{ open one end} \qquad (2)$$

where v is the speed of sound and L_{eff} is the effective length of the vibrating air column. The lowest mode of an open tube corresponds to a half-wavelength with a pressure maximum in the middle of the tube length, while the lowest mode of a tube closed at one end corresponds to a quarter-wavelength with a pressure maximum at the closed end. The open/open and open/closed cases can be easily distinguished by noting the pattern of harmonics.

The speed of sound depends on the temperature and chemical composition of the gas filling the tube. For dry air at typical lab temperatures the speed varies from 344 m/s at 20 °C to 347 m/s at 25 °C. The gas inside an instrument being played is saturated with water at approximately 27 °C, and contains 4% CO_2 and 16% O_2, giving a sound speed of 346 m/s.

The effective length L_{eff} is longer than the physical length L of the tube because the standing wave can extend beyond an open end. Direct measurements show that, to a good approximation, an open end increases the acoustic length by an amount $L' = 0.3d$, where d is the diameter of the tube.

The effective length of a pipe can be chosen to match the fundamental frequency to any musical note, but it is not generally convenient to provide one pipe for each note in a portable instrument. (Pan pipes and organs are exceptions, although the latter are not usually considered mobile.) The solution, discovered several thousand years ago, is to acoustically shorten the tube by opening a hole in the side, thereby releasing pressure before the end. If the hole is big enough, there will be a pressure node at the hole position, effectively terminating the standing wave. Smaller holes should have a less drastic effect, but would still tend to suppress part of the standing wave and raise the fundamental frequency. By opening several holes in succession up the tube it is possible for a musician to play all or most of the notes of a chosen scale. Since the size and position of the tone holes evidently determine the ability to play in tune, you will need to study this phenomenon.

We close this section with an elementary description of musical scales, for the benefit of non-musicians. The basic interval is an octave, which is a factor of two in frequency. In western music the octave is subdivided in such a way that after stepping off 12 intervals the frequency ratio is exactly 2:1. Since pairs of tones which have frequency ratios of certain small integers, like 3:2 or 5:4, make pleasant chords it is desirable that those ratios be available among the chosen frequencies. It is also advantageous if any note can be used as the base, while making all the desired ratios available. These requirements are mutually exclusive, so one of several compromises is needed. The usual modern choice is called equal temperament, in which successive notes have frequency ratios of $\sqrt[12]{2}:1$. Octaves are then exactly a factor of two, and the other intervals are considered close enough. For reference, both the frequency ratios and the actual frequencies for one octave from C_5, the lowest note on a common form of recorder, are tabulated below.

Table I Frequency ratios and frequencies for the equal-tempered scale from C_5 to C_6. Solid circles indicate the tone holes to be closed with thumb and fingers to play each note on a recorder.

note	ratio	frequency	LT	L1	L2	L3	R1	R2	R3	R4
C	2.0000	1046.50	●	○	●	○	○	○	○/○	○/○
B	1.8877	987.76	●	●	○	○	○	○	○/○	○/○
A#	1.7818	466.16	●	●	○	●	●	○	○/○	○/○
A	1.6818	880.00	●	●	●	○	○	○	○/○	○/○
G#	1.5874	830.61	●	●	●	○	●	●	○/●	○/○
G	1.4983	783.99	●	●	●	●	○	○	○/○	○/○
F#	1.4142	740.00	●	●	●	●	○	●	●/●	○/○
F	1.3348	698.43	●	●	●	●	●	○	●/●	●/●
E	1.2599	659.24	●	●	●	●	●	●	○/○	○/○
D#	1.1892	622.25	●	●	●	●	●	●	○/●	○/○
D	1.1225	587.35	●	●	●	●	●	●	●/●	○/○
C#	1.0595	554.38	●	●	●	●	●	●	●/●	○/●
C	1.0000	523.25	●	●	●	●	●	●	●/●	●/●

EXPERIMENTAL PROCEDURE

The experimental program has two phases: Measurement of the external acoustics of a typical recorder, and more detailed measurements on a model system.

1. Recorder acoustics

The measuring apparatus consists of a pressure-sensitive microphone connected to a computer running LoggerPro. The file Sound_spec sets up the program to produce a plot of microphone signal vs time, and a plot of intensity vs frequency derived from the Fourier transform of the signal. The required connections are shown in Fig. 2.

Cover all the tone holes on the recorder and blow gently through the mouthpiece to generate the lowest note. Use tape on the holes if you need a spare hand to operate other equipment. You will probably notice that if the input air flow is above a critical level the sound will abruptly increase in pitch. Performers use this technique to reach higher notes, but for testing purposes you want to remain in the low-flow regime.

Once you can reliably generate the lowest C, put the microphone near the recorder and use Sound_spec to obtain a frequency spectrum of the tone. To get the cleanest results, try to keep the amplitude constant for the full duration of the recording. You should also check a spectrum captured without sounding the instrument, so you can ignore any peaks due to background noise in the room.

Using the Examine tool or x = ? icon, record the frequencies of the fundamental and first few harmonics for the low C. Is the pattern of harmonics more consistent with a tube open at one end or at both ends?

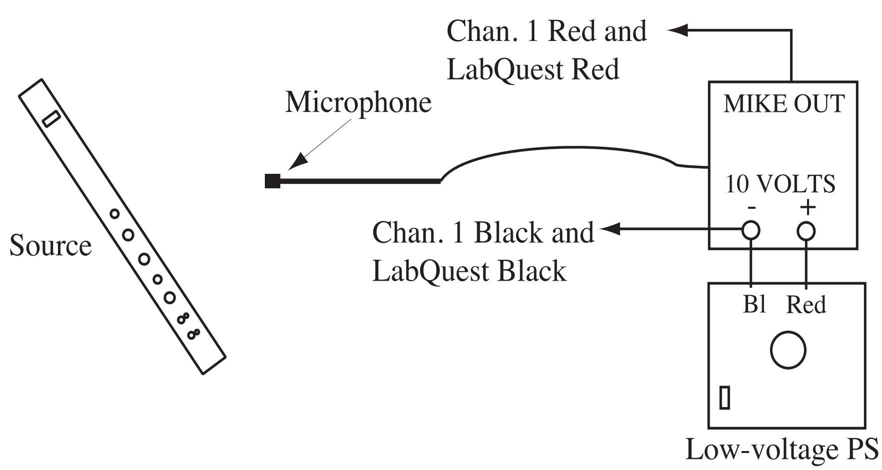

Fig 2. Wiring diagram for sound spectrum acquisition.

To see the effect of the tone holes, find the fundamental frequencies as you acoustically shorten the tube by successively opening all the holes from the foot of the instrument (not the air input end). Measure the distance from the knife edge to the highest open hole to approximate the physical length of the shortened tube.

Analyze your data by finding L_{eff} at each fundamental frequency and then noting how $L' = L_{eff} - L$ changes with the number of open holes. Are your results with no holes open consistent with the end correction for two open ends? How does L' change as you open holes? Later, you can compare the measurements with those for the model system to deduce the effect of the knife-edge opening.

The fingering chart also suggests that the effect of tone holes is more complicated than just shortening the tube. Note that you generally move up the scale by opening successive holes from the bottom, but there are exceptions. F is produced with only the third hole open, whereas E and G have the first two and first four open, respectively. Some other notes require uncovering only one of the double holes at the bottom of the instrument. Check some examples to see that these fingerings do make a difference. You will later want to see if these effects can also be found, and explained, in your model system.

2. Recorder model

The model recorder consists of a large plastic pipe with holes spaced along its length, to simulate the main tube. Instead of a turbulent air jet and edge, the system is driven by sine waves from a small loudspeaker placed near one end. This is a major simplification, since the excitation is easily controlled and will not be affected by the oscillating pressure in the air column.

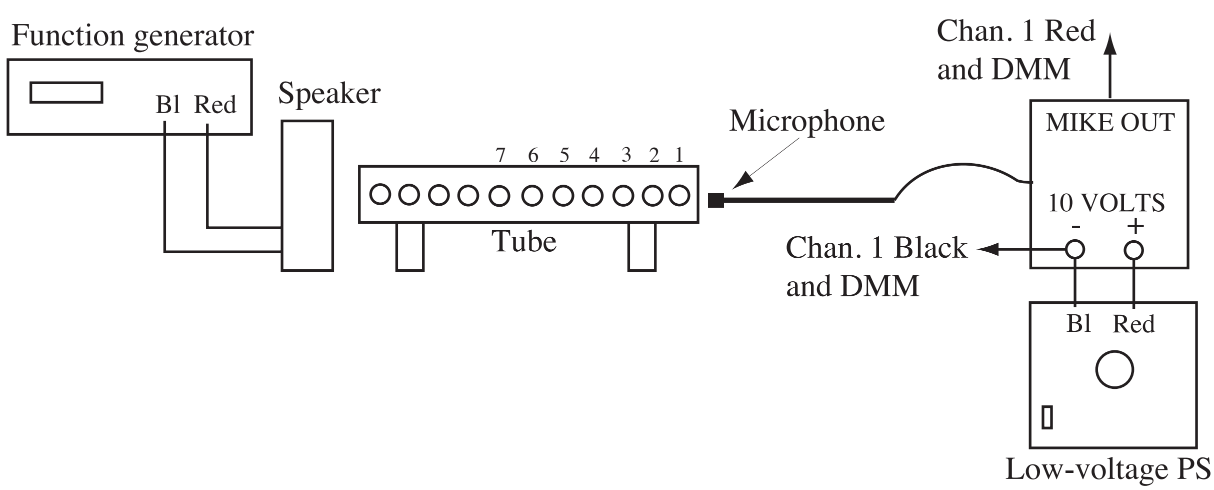

Fig 3. Arrangement of equipment for measurement of tube resonances.

The microphone signal can be displayed on the oscilloscope. The Measure > Amplitude function on the scope or AC volts on a DMM can be used for quantitative amplitude measurements. (Disconnect the LabQuest interface to avoid electrical interference with the DMM.) The microphone is small enough that it can be inserted into the tube for internal pressure measurements without significantly shifting the mode frequencies.

The first test of the model system is to determine the frequencies of the first few modes for the tube when all the side holes are closed with masking tape. To do this, put the microphone near the end opposite the speaker, and find the frequencies for which the acoustic signal is maximum. Be sure to keep the amplitude low, to avoid overloading the microphone or speaker.

You can also check that the pressure amplitude varies as expected along the length of the tube. Set the function generator for the fundamental resonance, and observe the relative amplitude as you slide the microphone along the inside of the tube. Do you get a single maximum near the middle, as expected? Are the amplitude patterns for one or two higher modes also as expected? If you externally trigger the scope from the TTL output you can see the relative phase of the oscillations as you cross a node.

Now you can check the effect of the simulated tone holes. As you did with the recorder, find the fundamental frequencies as you acoustically shorten the tube by successively opening all the holes, starting from the end opposite the speaker. You can take L to be the distance from the speaker end to the closest open hole to approximate the physical length of the shortened tube. Use your data to compute L' for each fundamental. Do you get results similar to those for the real instrument?

You should also see if the standard fingerings shift the fundamental as claimed. Referring to the fingering chart, opening the four lowest holes on a recorder would produce a G. Leaving only holes 1 and 4 open should slightly lower the frequency to F#. The actual frequency ratios won't be correct, but the effect should be evident.

The effect of hole size can be observed in a couple of ways. Check to see that opening 1, 1 1/2 and 2 holes from the bottom does shift the fundamental frequency up, as suggested by the fingerings for D, D# and E. A more direct test is done by comparing fundamental frequencies with the third hole open, as for an F on the real recorder, or only half open. If your results are as expected, it suggests that the tube is a good test system for hole size and placement.

By making internal pressure measurements along the length of the tube you can better understand the results you have gotten. Starting with all holes closed and the function generator set to the fundamental, measure and plot the amplitude vs position along the tube. It is quickest to enter the position and voltage readings directly into Graph.

Now open the 4 holes farthest from the speaker, as you would to get a G, set the frequency generator to the new resonance, and plot amplitude vs position on the same graph.

Comment on the differences between the two plots, and explain how opening a hole tends to raise the fundamental resonance, but not as much as actually cutting off the tube. Normalizing the data to the same maximum shows the deviations in the open parts more clearly. Fitting a quadratic function to the amplitude data points within the closed part of the tube may also help. (There is no theoretical basis for the quadratic but it provides a simple extrapolation of the single-peak fundamental mode.)

Similarly, make an amplitude-position plot to show the effect of closing holes below the highest open one. The fingerings for G and F# make a good comparison. What is the effect on the pressure distribution of closing holes below the highest open one?

Use your amplitude-position results to explain the variation in L' as holes are progressively opened in the model system. Comparing the variation of L' for the recorder with that for the model, can you infer an L' contribution for the input end of the recorder?

REPORT

Your report should document the tests you made and any conclusions you can draw about the acoustical behavior of pipes with holes. Which aspects of the recorder does your model system reproduce? Which are missing from the model?