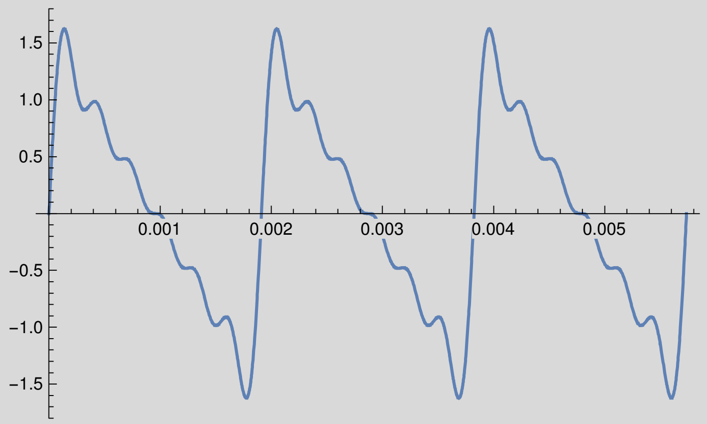
# Logarithmic Scales in Acoustic Spectra

By José Luis Gómez-Muñoz
http://homepage.cem.itesm.mx/jose.luis.gomez/

## Sawtooth wave

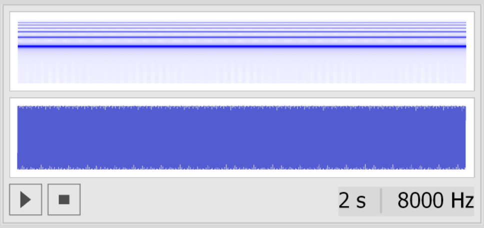
These are three periods of an approximation to a sawtooth wave:

```
Plot[Sin[2*Pi*523*t] + 1/2*Sin[2*Pi*1046*t] +
  1/3*Sin[2*Pi*1569*t] + 1/4*Sin[2*Pi*2092*t] +
  1/5*Sin[2*Pi*2615*t] + 1/6*Sin[2*Pi*3138*t], {t, 0, 3/523}]
```



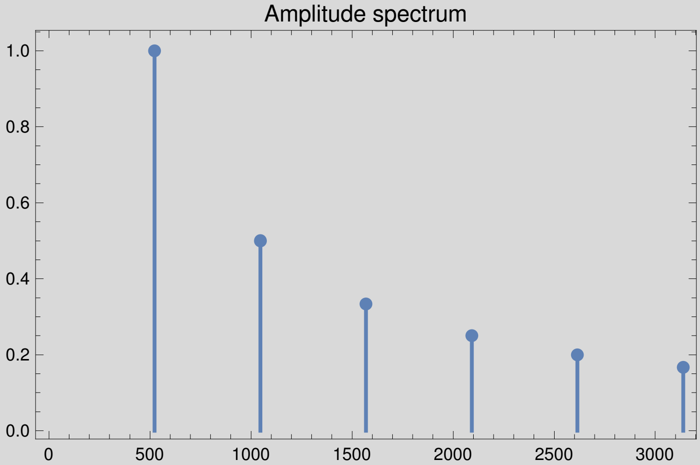

We can listen to the wave if we replace the command **Plot** with the command **Play** and the interval **{t,0,3/523}** (three period) with the interval **{t,0,2}** (two seconds), as shown below (if you are reading this document in *Mathematica* or the *CDFPlayer*, press the button ▶ in the result of the calculation below):

```
Play[Sin[2*Pi*523*t] + 1/2*Sin[2*Pi*1046*t] +
  1/3*Sin[2*Pi*1569*t] + 1/4*Sin[2*Pi*2092*t] +
  1/5*Sin[2*Pi*2615*t] + 1/6*Sin[2*Pi*3138*t], {t, 0, 2}]
```



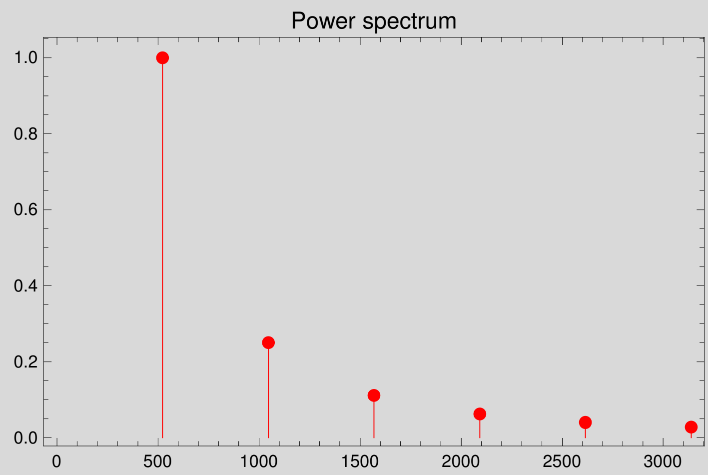

We can visualize the amplitude-spectrum of this wave with the command ListPlot. Remember that every point is made of the frequency and the corresponding amplitude, therefore the expression $\frac{1}{3}$ `* Sin[2 * Pi * 1569 * t]` becomes the point $\{1569, \frac{1}{3}\}$, and so on, as shown below:

```
ListPlot[
 {{523, 1}, {1046, 1/2}, {1569, 1/3}, {2092, 1/4}, {2615, 1/5}, {3138, 1/6}},
 Filling → Axis, FillingStyle → Thick,
 AxesOrigin → {0, 0}, Frame → True, PlotRange → All,
 PlotLabel → "Amplitude spectrum"
]
```



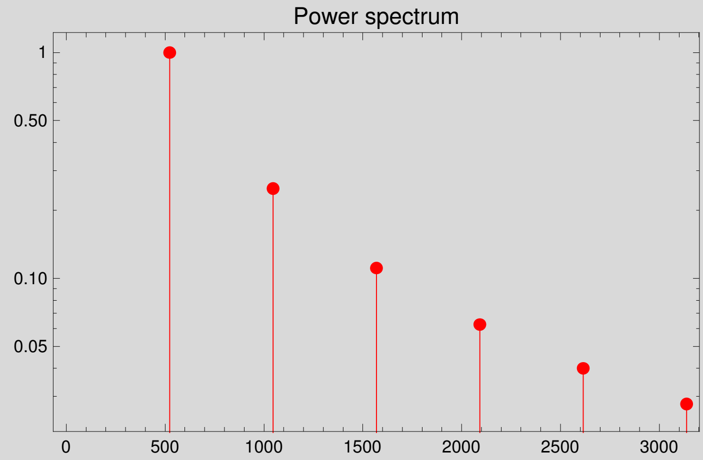

We can also visualize the **power-spectrum** of this wave with the command ListPlot. Notice that the important difference with the previous plot is that the amplitudes are squared: $1^2$, $(\frac{1}{2})^2$, $(\frac{1}{3})^2$, ...

```
ListPlot[
 {{523, 1^2}, {1046, (1/2)^2}, {1569, (1/3)^2},
  {2092, (1/4)^2}, {2615, (1/5)^2}, {3138, (1/6)^2}},
 Filling → Axis, FillingStyle → Red, PlotStyle → Red,
 AxesOrigin → {0, 0}, Frame → True, PlotRange → All,
 PlotLabel → "Power spectrum"
]
```

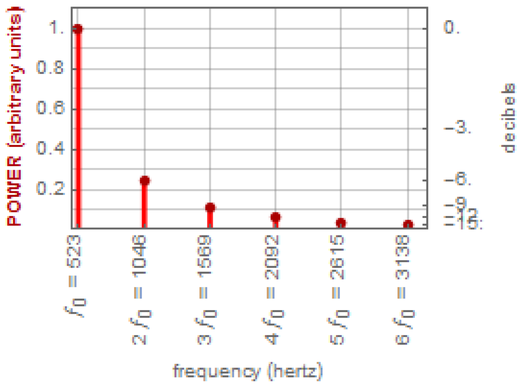


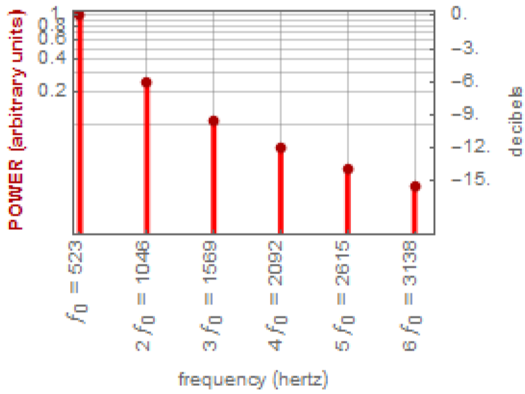

Notice that the spectrum above shows very small values together with large ones. This usually happens in Acoustics, and the details of the small amplitudes can be very important. Therefore we will change to another kind of plot, where small values and large values can be seen at the same time. Replace **ListPlot** with **ListLogPlot**, and replace **AxesOrigin→{0,0}** with **AxesOrigin→{0.01,0.01}**, and you obtain the following logarithmic plot for the same spectrum:

```
ListLogPlot[
 {{523, 1^2}, {1046, (1/2)^2}, {1569, (1/3)^2},
  {2092, (1/4)^2}, {2615, (1/5)^2}, {3138, (1/6)^2}},
 Filling → Axis, FillingStyle → Red, PlotStyle → Red,
 AxesOrigin → {0.01, 0.01}, Frame → True, PlotRange → All,
 PlotLabel → "Power spectrum"
]
```




Here we have the two graphs, they are for the **same** waveform, they are actually **the same spectrum**, see the power values and the decibel values:

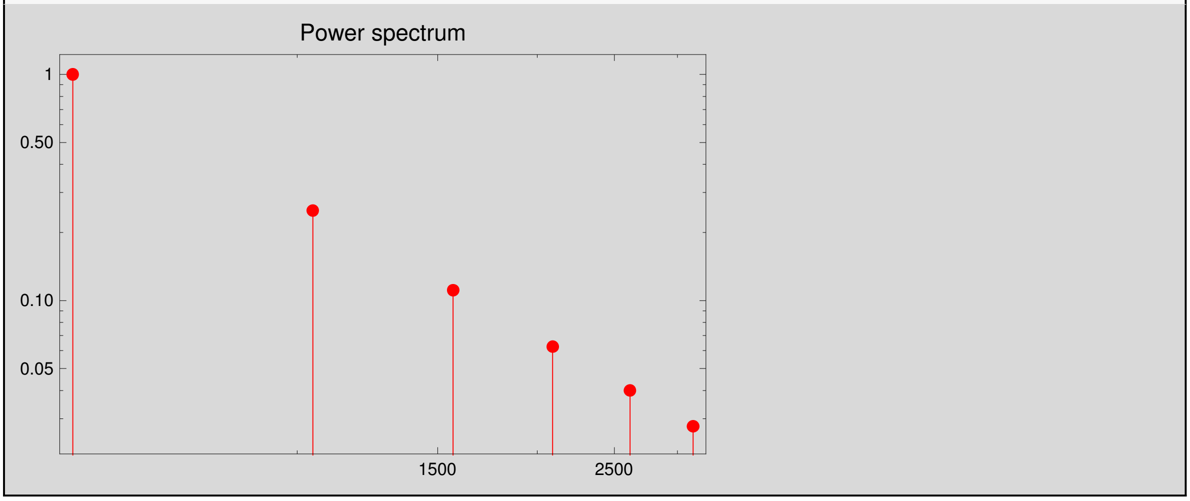


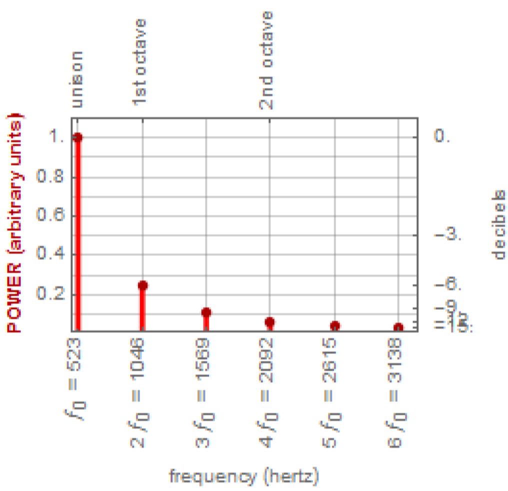

Using a logarithmic scale is similar to the effect of this mirror, so that the small peaks are enlarged and can be compared with the large peaks in the spectrum

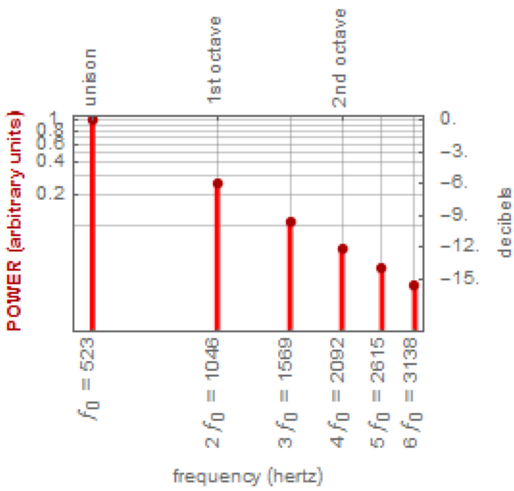
Furthermore, we can also have a logarithmic scale in the horizontal axis. In order to obtain that graph, replace the command **ListLogPlot** with the command **ListLogLogPlot** and set **AxesOrigin→{450,0.01}**:

```
ListLogLogPlot[
 {{523, 1^2}, {1046, (1/2)^2}, {1569, (1/3)^2},
  {2092, (1/4)^2}, {2615, (1/5)^2}, {3138, (1/6)^2}},
 Filling → Axis, FillingStyle → Red, PlotStyle → Red,
 AxesOrigin → {450, 0.01}, Frame → True, PlotRange → All,
 PlotLabel → "Power spectrum"
]
```



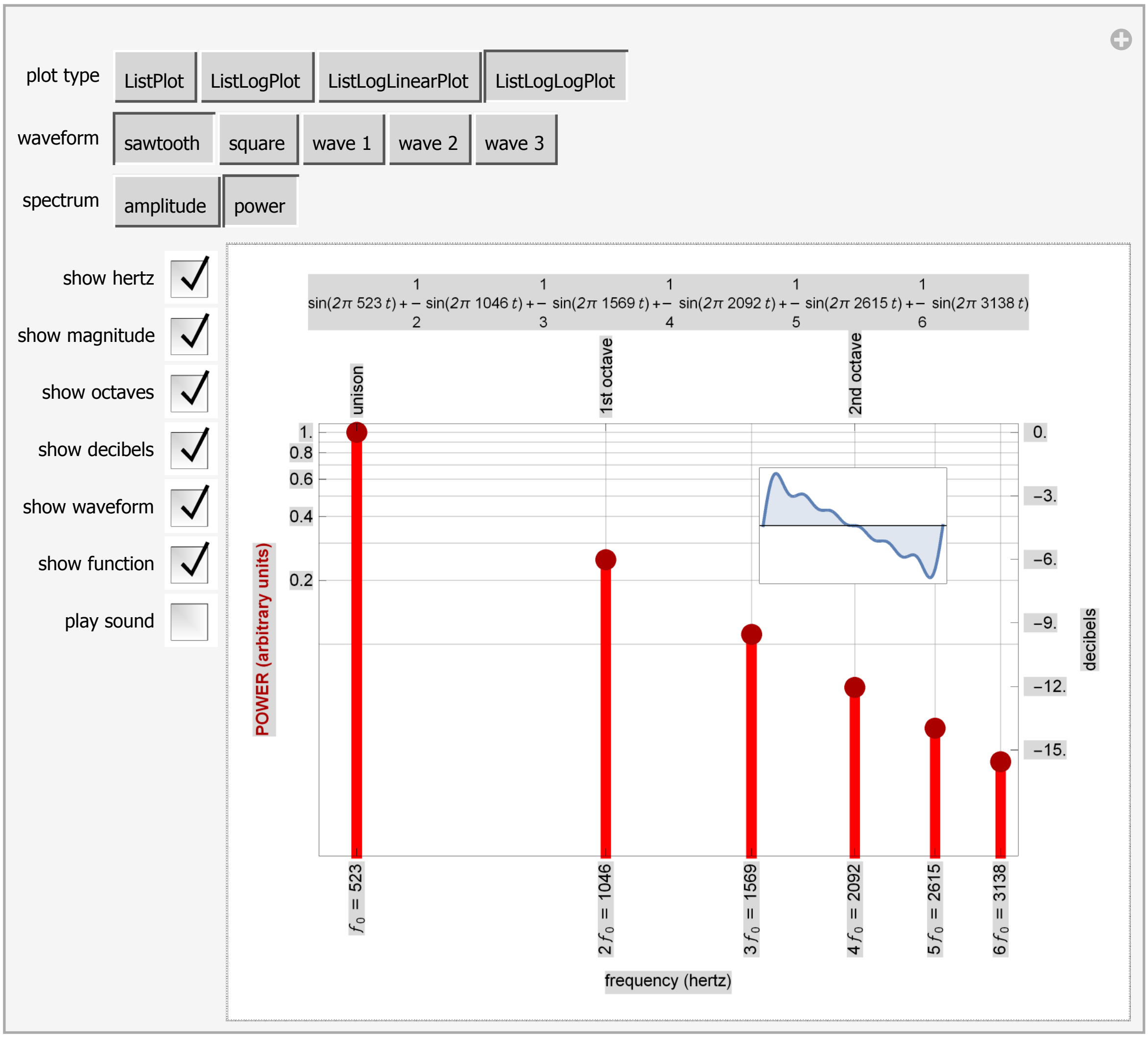

Here we have the two graphs, they are for the **same** waveform, they are actually **the same spectrum**, check the coordinates of each point in both of them, also see the decibel and octave scales:




# Interactive Demonstration of Logarithmic Scales

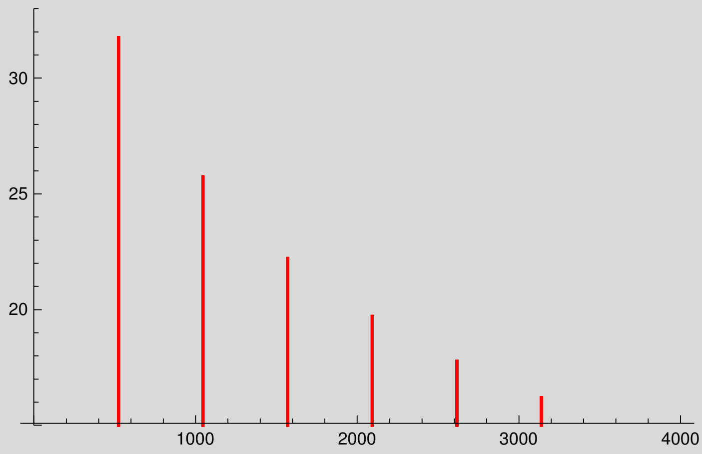
Please **enable dynamic content** in *Mathematica* or the *CDFPlayer* in order to be able to interact with the following demonstration:




# The Periodogram command

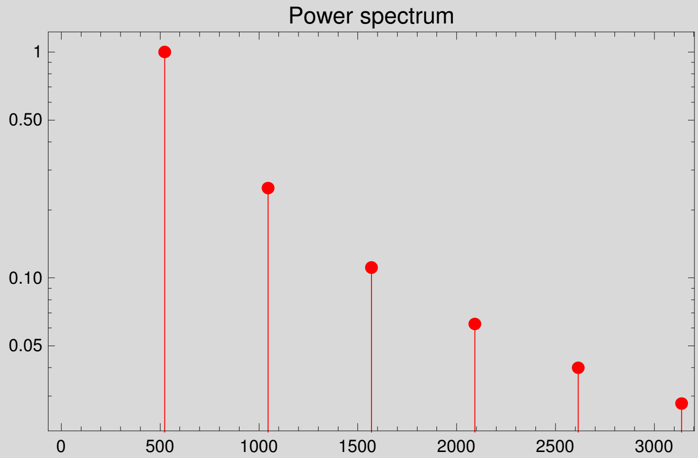
*Mathematica*'s command Periodogram can be used to obtian the power spectrum, with a logarithmic vertical axis (decibels) and a linear horizontal axis (Hertz)

```
Periodogram[Play[Sin[2 * Pi * 523 * t] + 1/2 * Sin[2 * Pi * 1046 * t] +
    1/3 * Sin[2 * Pi * 1569 * t] + 1/4 * Sin[2 * Pi * 2092 * t] +
    1/5 * Sin[2 * Pi * 2615 * t] + 1/6 * Sin[2 * Pi * 3138 * t], {t, 0, 2}],
 PlotRange → {15, 33},
 PlotStyle → Red]
```




Compare the previous graph, with the command Periodogram, and the graph below, with the comand ListLogPlot:

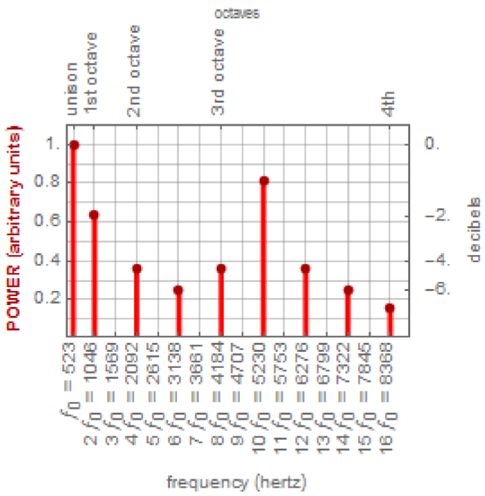
```
ListLogPlot[
 {{523, 1^2}, {1046, (1/2)^2}, {1569, (1/3)^2},
  {2092, (1/4)^2}, {2615, (1/5)^2}, {3138, (1/6)^2}},
 Filling → Axis, FillingStyle → Red, PlotStyle → Red,
 AxesOrigin → {0.01, 0.01}, Frame → True, PlotRange → All,
 PlotLabel → "Power spectrum"
]
```

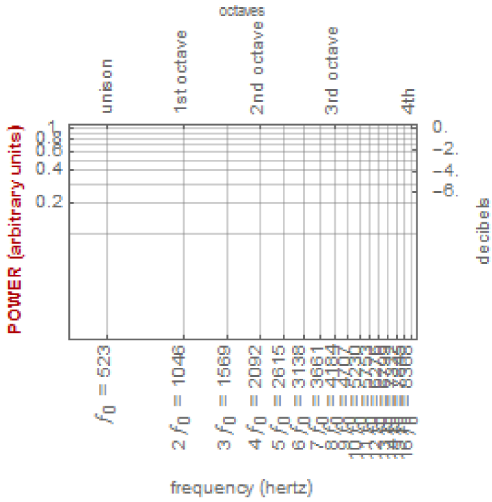


# Exercises

## Exercise 1

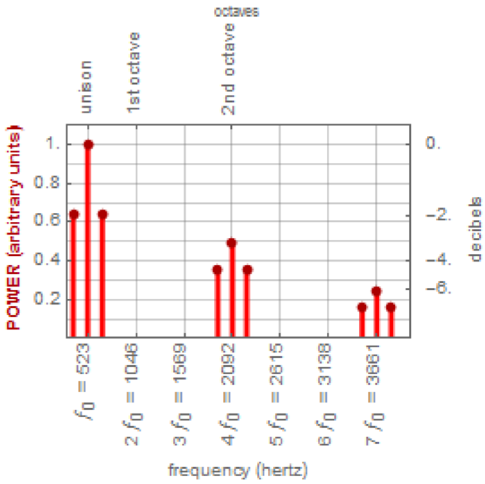
Graph this spectrum with logarithmic scales:

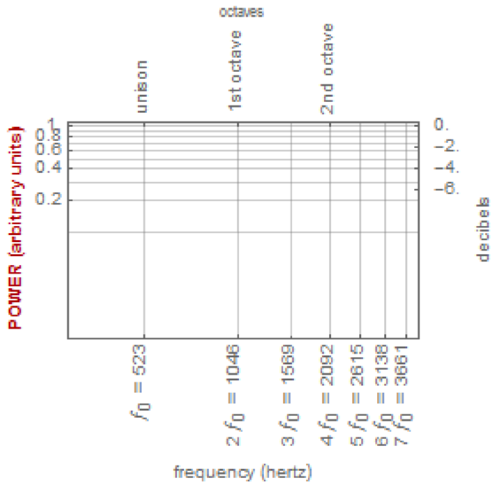



## Exercise 2

Graph this spectrum with logarithmic scales:




Author: José Luis Gómez-Muñoz http://homepage.cem.itesm.mx/jose.luis.gomez/

```
{DateString[], $Version}
```

```
{Wed 14 Jan 2015 10:43:58,
 10.0 for Microsoft Windows (64-bit) (June 29, 2014)}
```